Example

Vincent and Grantham[1] and Stengel[2] both provide some analytical examples of parameter optimal control. The following numerical example demonstrates the use of parameter optimal control in trajectory optimization.

A trajectory optimization problem can be converted into a parameter optimal control format with different control parameterizations. There are three ways of parameterizing control functions. The most common one is to approximate a control function by piecewise constant (Goh and Teo[7]) or general spline functions. Control functions can also be approximated by polynomials or other functions of time. The validity of this parameterization is guaranteed by the Weierstrass theorem.[8] In addition, one can assume special parameterization forms for controls.

The equations of an ascending rocket are given[2] as

$$\dot{v} = \frac{T}{m} \cos \alpha - g \sin \gamma \tag{19}$$

$$v\dot{\gamma} = \frac{T}{m} \sin \alpha - g \cos \gamma \tag{20}$$

where v is the flying speed in ft/s, γ is the flight path angle defined with respect to the horizontal plane, and α is the angle between the thrust vector and the velocity vector. The control angle $\alpha(t)$ is approximated by a polynomial function of time

$$\alpha(t) = \alpha_0 + \alpha_1 t + \cdots = \sum_{k=0}^{N_p - 1} \alpha_k t^k \tag{21}$$

The problem then is to determine parameters α to minimize

$$I = \left(\gamma_f - \frac{\pi}{3}\right)^2 + \int_0^{t_f} \alpha^2(t)\, \mathrm{d}t \tag{22}$$

The initial conditions are $v(0) = 100$ ft/s and $\gamma(0) = \pi/2$. Other parameters are $t_f = 10$ s, $T = 10{,}000$ lbs, $m = 20$ slugs, and $g = 32$ ft/s^2.

The second-order algorithm is used with the initial guesses of $\alpha_k = 0$, $k = 0, 1, \ldots, N_p$. Figure 1 shows the reductions in the optimal cost as the number of parameters increases. Figure 2 compares $\alpha(t)$ histories for cases of up to seven parameters. A further increase in the number of parameters does not decrease the cost very much.

Conclusions

This Note employs two basic approaches and presents the necessary and sufficient conditions for an unconstrained parameter optimal control problem. Two numerical solution methods are also given. An example problem is used to demonstrate the use of parameter optimal control and polynomial parameterization in trajectory optimization. Use of the gradient expressions in numerical solutions avoids the large number of integrations needed in digital differentiations.

References

[1]Vincent, T. L., and Grantham, W. J., *Optimality in Parametric Systems,* Wiley, New York, 1981, Chap. 6.

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[3]Rogers, R. M., "Parameter Optimal Control Solution Technique for Nonlinear Systems Using Influence Functions," *Proceedings of the AIAA Guidance, Navigation, and Control Conference* (Hilton Head, SC), AIAA, Washington, DC, 1992, pp. 1163–1168 (AIAA Paper 92-4552).

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Book Announcements

KAUFMAN, H., BAR-KANA, I., and SOVEL, K., ***Direct Adaptive Control Algorithms,*** Springer-Verlag, New York, 1994, 374 pages, $69.00.

Purpose: This text contains an in-depth review of adaptive control techniques that have been developed by the authors over the last decade. The monograph presents a rigorous treatment of multi-input, multi-output control theory as well as a thorough discussion of algorithmic implementation features and applications.

Contents: Output model following; stability and positivity; nonlinear adaptive controllers; positive real analysis; parallel feedforward control; model reference adaptive control; robust redesign of adaptive algorithms; robustness considerations with feedforward in the reference model; design of MRAC systems; case studies.

ESTRADA, R., and KANWAL, R. P., ***Asymptotic Analysis: A Distributional Approach,*** Birkhauser, Cambridge, MA, 1994, 258 pages, $64.95.

Purpose: This reference provides a theoretical framework for asymptotic expansions encountered in the solution of a large class of problems in engineering and physics.

Contents: Asymptotic series; algebraic and analytic operations; space of distributions; regularizations; distributional derivatives; tempered distributions; expansion of oscillatory kernels; multidimensional generalized function; series of Dirac delta functions.

GRAY, W., LEIJNSE, A., KOLAR, R. L., and BLAIN, C. A., ***Mathematical Tool for Changing Spatial Scales in the Analysis of Physical Systems,*** CRC Press, Boca Raton, FL, 1993, 323 pages, $39.95.

Purpose: This book contains comprehensive introduction to fundamental mathematics for changing spatial scale in physical models. As such, it is a welcome addition to the growing literature applicable to homogenization as encountered in damage mechanics and mechanics of composite materials.

Contents: Scale and derivation of balance laws; generalized functions of curves; surfaces and volumes; integration scales; integration region selection with generalized functions; derivation of averaging theorems.

DE BOOR, C., HOLLIG, K., and REIMANSCHNEIDER, S., ***Box Splines,*** Springer-Verlag, New York, 1994, 200 pages, $34.00.

Purpose: This book describes the mathematical theory of multivariable splines that encompasses many results that have, until now, only been available in journal articles.

Contents: Definitions of box splines: analytic, geometric, inductive; recurrence relations; zonotopes; linear algebra of box splines; quasi-interpolation and approximation powers; cardinal interpolation; difference equations; cardinal splines; wavelets; discrete box splines; subdivision algorithms.